Difficulties encountered by students to recognize « lines » and « planes » in space

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Context

- Thesis first step

Purpose

Set up a diagnosis of a mathematics course in university about lines and planes equations in space.

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$\Rightarrow$ Deduce some teaching specificities and study difficulties encountered by students.

Plan

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Teaching sequence

- Chapter following the study of lines equations in plane.
- Walkthrough :
  - incomplete planes equations.
  - $ax + by + cz = d$ planes equations.
  - parametrical lines equations in space.
  - cartesian lines equations system in space.

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Describe the set geometrically
$B := \{(x_1, x_2, x_3) \in \mathbb{R}^3 |\ (x_1, x_2, x_3)$ an orthogonal vector to $(-1, -3, 0)\}$.

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$\Rightarrow$ This flexibility need to be developed during learning phase (Artigue, Chartier & Dorier, 2000).

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Section chosen : mathematics (24 students)

Plan

1 Analysis of the studied teaching

2 The students' practices

3 Conclusion

Expected practices

Exam, Q11, 2014

Let the set $S_2 := \{(x, y, 0) \in \mathbb{R}^3 | x, y \in \mathbb{R}\}$.
Describe this set geometrically and represent it graphically. Explain your walkthrough.

- The plane is described from a set point of view.
- We are expecting students to recognize the plane and describe it with an equation.
- This equation is given from a cartesian ($z = 0$) or a parametrical (Oxy) point of view.
- The plane will be represented in space.

$\Rightarrow$ Switching of points of view and algebrical and graphical registers conversion are needed.

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- Graphical representation : 54% of the students are able to represent it as a plane correctly. Answers distribution is :
  - 12.5 % of the students from a cartesian point of view.
  - 25% of the students from a parametrical point of view.
  - 16.5% of the students used both points of view.

Actual practices (2/2)

Results

- Most of the students seem to master switching points of view.
- Going from an algebrical register to a graphical register seem to be difficult for students having described the plane from a cartesian point of view.
- 41.5% of the students seem to use some flexibility between points of view and are able to switch between registers.

Plan

Conclusion

- Object recognition from a specific point of view seems to be gained for most of the students.
- Switching between set/cartesian or set/parametrical points of view is mastered.
- Difficulties related to register changing are proves of a lack of meaning accorded to equations for some of the students.
- Students majority hasn't developed the attented flexibility.
- Perspectives :
    - study of the difficulties related to the use of equations.
    - a deeper exercices and tests analysis to identify flexibilty related difficulties.